INFLUENCE OF END IMPEDANCES ON THE NATURAL FREQUENCIES OF AN ACOUSTIC RESONATOR

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An acoustic resonator, consisting of a tube closed at one end by a radiator, and at the other by a sound receiver, is often used for investigating the velocity of propagation of sound in gases. The influences of the radiator impedance Z_0 and receiver impedance Z_L on the natural frequencies of the resonator are usually allowed for by introducing an effective resonator length, determined from calibration experiments. This does not always lead to correct results, however, since Z_0 and Z_L, and hence the effective length, depend on the experimental conditions and the type of gas being measured, as well as on the construction of the radiator and receiver.

A method is described below, such that the influence of the end impedances can be taken into account when calculating the natural frequencies of the resonator.

The natural frequencies f_n of an acoustic resonator are determined by the condition [1]

$$2k_n L - 2\delta_0 - 2\delta_L = 2\pi n, \qquad k_n = \frac{2\pi f_n}{a}, \qquad n = 1, 2, 3 \ldots . \tag{1}$$

Here a is the velocity of sound, L the resonator length, and $2\delta_0$ and $2\delta_L$ are the phase shifts in the end impedances. The phase shifts 2δ are determined by the end impedances and the properties of the gas filling the resonator [1]

$$\operatorname{tg} 2\delta = \frac{2Y}{Y^2 + R^2 - 1}, \quad R = \frac{\operatorname{Re}(Z)}{S\rho a}, \quad Y = \frac{\operatorname{Im}(Z)}{S\rho a}. \tag{2}$$

Here S is the area of the resonator cross section, ρ the gas density, and Z the mechanical impedance of the sound radiator (receiver).

Since, in the case of electromagnetic and condenser microphones, $\operatorname{Im}(Z) \gg \operatorname{Re}(Z)$ [2, 3] at frequencies of the order of 1000 Hz, reasonably well away from the fundamental resonance, the mechanical impedance of the radiator (receiver) can be written as

$$Z \approx j\omega M_0 - j\frac{E_0}{\omega} + j\omega M_p - j\frac{E_p}{\omega}. \tag{3}$$

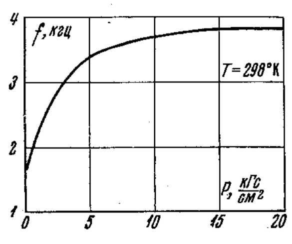

Fig. 1

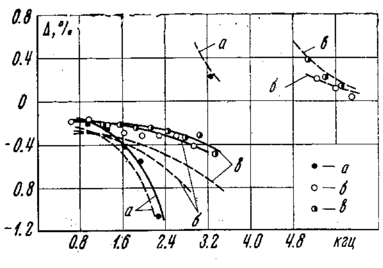

Fig. 2

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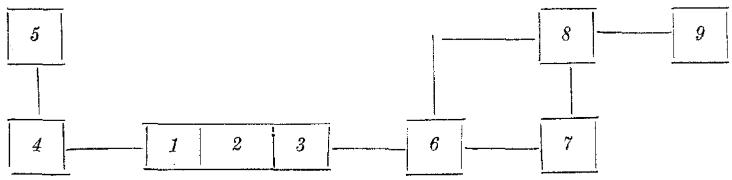

Fig. 3

The first two terms here represent the imaginary part of the membrane mechanical impedance [4], while the last two are corrections allowing for the influence of the gas, p being the gas pressure in the resonator. The coefficients M and E, which depend on the radiator (receiver) construction and on the molecular weight and temperature of the gas, can be found from the resonance condition Im(z) = 0 on the basis of the frequencies f_{01} and f_{02} of mechanical resonance of the radiator (receiver) measured at two different pressures p_1 and p_2

$$M = M_0 \left\{ \frac{1}{p_1} \left(\frac{f_{01}^2 - f_{00}^2}{f_{02}^2 - f_{01}^2} \right) - \frac{1}{p_2} \left(\frac{f_{02}^2 - f_{00}^2}{f_{02}^2 - f_{01}^2} \right) \right\}$$

$$E = E_0 \left\{ \frac{1}{p_1} \frac{(f_{01}^2 - f_{00}^2) f_{02}^2}{(f_{02}^2 - f_{01}^2) f_{00}^2} - \frac{1}{p_2} \frac{(f_{02}^2 - f_{00}^2) f_{01}^2}{(f_{02}^2 - f_{01}^2) f_{00}^2} \right\} \tag{4}$$

$$f_{00}^2 = \frac{1}{(2\pi)^2} \frac{E_0}{M_0}.$$

By using (2)-(4), the radiator (receiver) mechanical impedance and resonant frequency can be calculated, together with the natural frequencies of the resonator. The theoretical curve in Fig. 1 of the resonant frequency of an electromagnetic rediator (receiver) against the gas pressure in the resonator is in good agreement with the experimental data.

If the radiator and receiver have the same construction ($Z_0 = Z_L = Z$), the relative deviation of the resonator natural frequencies f_n from the resonant frequencies $f_n^0 = 2nL/a$, corresponding to $z = \infty$, is

$$\Delta = \frac{f_n - f_n^\circ}{f_n^\circ} = \frac{2\delta}{\pi n} \tag{5}$$

Frequency curves of the relative deviations in the resonator natural frequencies, due to the influence of the end impedances, are given in Fig. 2. The theoretical (broken) and experimental (continuous) curves refer to a resonator of 0.03 m diameter and 0.409 m length with the respective pressures a) 1, b) 5, and c) 20 kgf/cm^2. The following quantities were used for calculating the end impedances: $M_0 = 6.6 \cdot 10^{-4}$ kg, $f_{00} = 1300$ Hz, $f_{01} = 2400$ Hz, $f_{02} = 3700$ Hz, $p_1 = 1$ kgf/cm^2, and $p_2 = 10$ kgf/cm^2.

It can be seen from Fig. 2 that the end impedances have the effect of considerably reducing the resonator natural frequencies in the range below the resonant frequency of the radiator (receiver), and of increasing the natural frequencies in the range above the radiator (receiver) resonant frequency. The measured values of the deviations are in satisfactory agreement with the theoretical data. The difference, espe-

TABLE 1. Velocity of Sound in Carbon Dioxide, m/sec

p kgf/cm²	1		5		10		20		30		40	
T °K	a	a_*	a	a_*	a	a_*	a	a_*	a	a_*	a	a_*
293	265.2	—	263.2	—	258.7	—	249.7	—	240.0	—	230.3	—
303	269.8	270.7	267.7	267.8	263.8	264.0	255.8	256.1	247.8	247.6	239.0	233.9
323	—	—	276.3	—	273.1	—	266.8	—	261.1	—	254.6	—
353	—		288.4	238.5	285.8	286.3	281.4	281.8	277.5	277.6	273.1	273.5
373	—	—	296.0	—	293.6	—	290.4	—	287.0	—	233.5	—
413	—	—	310.4	340.9	308.4	309.8	306.3	307.9	304.1	305.3	302.3	303.2

cially in the region close to the radiator (receiver) mechanical resonance would seem to be explained by the influence of the resistive component of the impedance, which was discounted in the above theory.

A block diagram of the experimental set-up is given in Fig. 3. Here 1 is the radiator, 2 the resonator, 3 the receiver, 4 the sound generator, 5 the frequency meter, 6 an amplifier, 7 a limiter, 8 a synchronous detector, and 9 a microammeter. The synchronous detector enabled the resonant frequencies to be determined accurately (±0.1 Hz) with a low radiator power and low receiver sensitivity. Due to the reduced sensitivity, it was possible to increase the mechanical impedance of the electromagnetic radiator (receiver).

Measurement data on the velocity of sound in carbon dioxide are given in Table 1. For comparison, the velocities a_* evaluated from the equation of state [5] are also given.

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